Birzeit University
Faculty of Engineering
Department of Electrical Engineering
Engineering Probability and Statistics ENEE 331
Problem Set (4)
Multiple Random Variables

Instructor: Dr. Wael Hashlamoun

1) For a certain commodity which you buy, you can make either a $500 profit with probability 0.5 when you sell it, or $200 with probability 0.3 or lose $100 with probability 0.2.

a. Find the mean and variance of your net profit if you sell one item.
b. Suppose you sell 80 items separately and independently, find the mean and standard deviation of your total net profit.

2) Two random variables X and Y are related by Y = aX + b, where X is a random variable with zero mean and unit variance.

a. Find the mean and variance of Y
b. Find the correlation coefficient between X and Y.

3) Let X and Y be random variables with a joint pdf $f_{X,Y}(x,y)=C$ for $0 \le X+Y \le 1$, $0 \le X \le 1$, $0 \le Y \le 1$

a. Find C so that this is a valid joint pdf
b. Find the marginal density functions of X and Y.
c. Are X and Y independent?
d. Find the conditional pdf of Y given X = 0.5

4) If X and Y are independent, normal random variables with E(X) = 10, Var(X) = 4, E(Y) = 0, and Var(Y) = 9.

a. Let T = X – Y, find the mean and variance of T
b. Let Z = XY, find the mean and variance of Z.

5) The random variables X and Y are independent and uniformly distributed in the interval (0,1). Find $P(Y \le \sqrt{X})$.

6) Let X be a uniformly distributed random variable on the interval $0 \le x \le 10$ and zero elsewhere and let Y be another uniformly distributed random variable on $0 \le y \le 20$ and zero elsewhere. Assuming that X and Y are independent, find

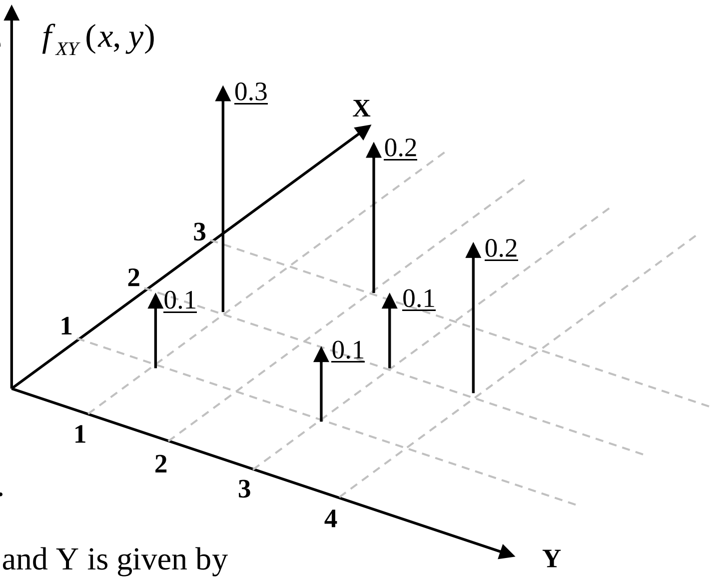
a. $P(X \le 4 \cap Y \le 8)$
b. E {X + Y}
c. E {XY}
d. Var (X + Y)

7) The lifetime of a structure **T** is a Gaussian distribution which is dependent on the strength of used concrete. B250 has μ = 35 years, σ = 10 years, whereas B300 has μ = 50 years, σ = 5 years.

a- If a structure with design period of 40 years will be designed, which concrete is better to be used?

b- For B300, find time in years at which the lifetime of the structure will exceed 95% of its design period.

8) For the joint density function shown in the figure, find the followings:



a- Marginal density functions of X and Y
b- P (X < 3)
c- P (Y ≥ 2)
d- P [X = x / (Y = 1)]
e- P (X ≥ Y)

9) Let X_1 and X_2 be independent normal random variables with means 23 and 4 and variances 3 and 1, respectively. Find the probability density function of $Y = 4X_1 - X_2$.

10) The joint pdf of two random variables X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} kxy & 0 \le x \le 2 \quad 0 \le y \le 3 \\ 0 & otherwise \end{cases}$$

a. Find the constant k so that this is a valid pdf.
b. Are X and Y statistically independent?
c. Find the expected value of the function g(X,Y) = 2X + 3Y
d. Find P(X + Y < 1), P(Y – X < 1).